ELECTRICAL CURRENT AND POTENTIAL DISTRIBUTION IN FLAT CHANNEL WITH POINT ELECTRODES

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We use the example of a flat channel with point electrodes to examine the combined effect of anisotropy and freezing-in. Expressions are obtained for the current and potential. The current and equipotential lines are calculated for various values of the Hall parameter and magnetic Reynolds number.

A large number of studies [1] have been devoted to determining the electric fields in channels with a moving electrically conducting medium. Most studies relate to the case $R_m \ll 1$ (R_m is the magnetic Reynolds number).

The current distribution with account for freezing-in for semiinfinite electrodes was found in [2] by the iteration method. The anisotropy of the conductivity occurring in gaseous media complicates considerably the current and potential distribution pattern in the channel [3]. We shall examine a flat channel with point electrodes in which a conductive medium is flowing. We assume that the velocity of the conducting medium in the channel is constant in time and independent of the coordinates. For a magnetic Reynolds number $R_m > 0$ the current will be carried downstream because of freezing-in. We assume that the external magnetic field is maintained constant. This means that the applied magnetic field remains uniform in the channel. Therefore we can examine separately the magnetic field of the current, particularly since the constant component of the field does not enter the expressions for the current

$$\operatorname{rot} \mathbf{H} = \frac{4\pi}{c} \mathbf{j} \tag{1.1}$$

The question of what the external magnetic field must be to provide the given gas velocity U = const is not examined. The distributions of the current density **j** and potential φ are assumed bivariate, the conductivity σ is assumed constant.

We direct the x axis along the centerline of the channel of unit height. Let the channel walls be the straight lines $y = \pm^1/_2$ and the electrodes located at the points $(0, \pm^1/_2)$. The magnetic field is directed along the z axis (see Fig. 1).

In this case the induction equation [4] has the form

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} - R_m \frac{\partial H}{\partial x} = 0 \tag{1.2}$$

Here dimensions are referred to the channel height, and the magnetic field is referred to the quantity $4\pi I/c$, where I is the total current.

Let us formulate the boundary conditions for the induction equation. The current density on the channel walls is given

$$y = \pm^1/_2, \; j_y = \delta(x) \tag{1.3}$$

where $\delta(x)$ is the first-order impulse function [5]. The conditions at infinity $H(-\infty) = 1$, $H(+\infty) = 0$ are equivalent to specification of the total current I.

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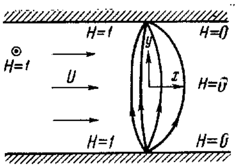

Fig. 1

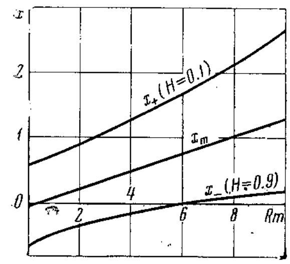

Fig. 2

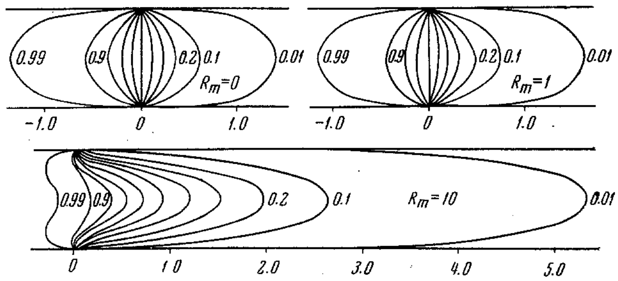

Fig. 3

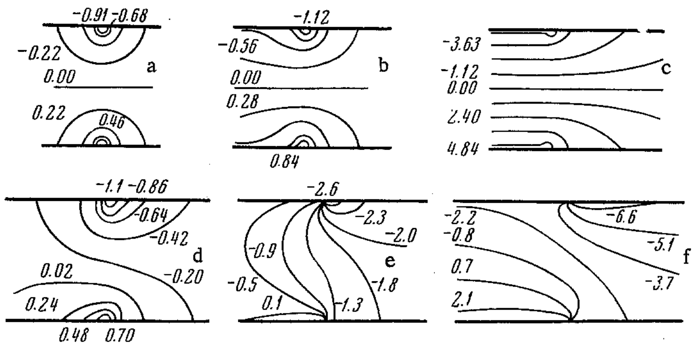

Fig. 4

The solution of (1.2) is obtained by the integral Fourier transform method and has the form

$$H = 1 - 4\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\lambda_n \cos(\lambda_n y)}{\mu_n(2\mu_n - R_m)} e^{\mu_n x}, \qquad x \leqslant 0$$

$$H = 4\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\lambda_n \cos(\lambda_n y)}{\nu_n(2\nu_n + R_m)} e^{-\nu_n x}, \qquad x \geqslant 0 \tag{1.4}$$

$$\lambda_n = (2n-1)\pi, \quad \mu_n = {}^1/_2\,[R_m + \sqrt{R_m^2 + 4\lambda_n^2}], \quad \nu_n = \mu_n - R_m$$

It is convenient to characterize the current shift by the quantity x_m, which is the moment of the current

$$x_m = \int_{-\infty}^{\infty} x j_y(x, 0)\, dx = \frac{R_m}{8}$$

[The integral is easily calculated using (1.1) and (1.4).]

Thus, if the distributed current is replaced by a concentrated equivalent current the coordinate of this current is proportional to R_m.

The boundaries of the region occupied by the current can be estimated using (1.4). This requires finding the roots of the equation H(x, 0) = const. For example, H(x) = 0.1 and H(x) = 0.9 show the boundaries of the region containing 80% of all the current (Fig. 2). For $R_m > 1$ the current is carried away strongly and for $R_m \sim 10$ the current at the channel centerline is beyond the electrodes entirely.

The complete current distribution pattern is shown in Fig. 3.

The distribution of the potential φ in the case of point electrodes is found by quadrature from Ohm's law and (1.4)

$$\frac{\sigma}{I}\varphi=\frac{\beta}{2}[H^2-(1+H_0)^2]-\int\limits_{-\infty}^{x}\frac{\partial H}{\partial y}dx-R_m(1+H_0)y \tag{1.5}$$

Here the Hall parameter β is defined from the magnetic field $4\pi c^{-1}I$ of the total current. The potential is assumed to vanish for $x \to -\infty$ and $y = 0$.

The equipotential lines $\varphi(x, y)$ = const shown in Fig. 4 were calculated using (1.5). For $\beta = R_m = 0$ the equipotential lines are perpendicular to the current lines (see Fig. 4a and Fig. 3), just as in the case of a source and sink located on opposite walls of the channel.

If $R_m > 0$, as the conducting medium travels in the current self-magnetic field there is induced an emf directed opposite the voltage applied to the electrodes (counter emf). The voltage on the electrodes must increase in order to assure a given current in the presence of the counter emf. This electric field increase is the larger, the larger the self-magnetic field. Where the field is large there is practically no current and the equipotential lines are parallel to the walls. Where the magnetic field abates and current flows the equipotential lines curve [see Fig. 3, Fig. 4b ($R_m = 1$), Fig. 4c ($R_m = 10$)].

In the case of an anisotropically conducting medium the current distribution does not depend on the Hall parameter β, but the potential distribution changes in this case. A longitudinal potential difference develops at the infinitely distant ends of the channel and prevents longitudinal current flow. In this case the equipotential lines acquire a characteristic slope [see Fig. 4d ($R_m = H_0 = 0$, $\beta = 0.6$), Fig. 4e ($R_m = H_0 = 0$, $\beta = 4$)].

We see from (1.5) that the potential difference at points which are symmetric relative to the x axis is independent of β. We note that in the case of electrodes of finite size the potential difference increases with increase of β [3].

This fact can be explained more clearly as follows. By virtue of the current distribution symmetry about the x axis the Hall electric field, proportional to j_y, will also be symmetric. Therefore the potential will increase by the same amount at symmetric points. The potential difference remains unchanged.

In the finite electrode case the current distribution is not symmetrical, the potential increases to a different degree at symmetric points, so that the potential difference increases.

We note that as a result of the Hall effect the electric field at the anode becomes higher than at the cathode (at symmetric points), i.e., the distance between the equipotentials (Fig. 4d) is less at the anode.

The presence of the self- and applied-external magnetic fields leads to the appearance of additional induced electric fields, as a result of which the equipotential lines take the form shown in Fig. 4f ($R_m = 1$, $\beta = 1$, $H_0 = 4$). The solution presented above is easily extended to the case of an arbitrary number of point electrodes.

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